Model risk and stress testing

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Based on **Claudio Albanese, S.C.** and **Stefano Iabichino**, ***A Darwinian Theory of Model Risk***, ssrn.3544862, and ***Reverse Stress Testing***, ssrn.3544866, working papers (2020).

Introduction

trade-specific pricing,
high-quality pricing models

$\xrightarrow{\text{2008 crisis}}$ netting-set CVA analytics,
low-quality pricing models

$\xrightarrow{\text{FRTB}}$ emphasis on ***model risk***

A Darwinian theory of model Risk

Reverse stress testing

Conclusions

Model risk for structured products

- bank buys convexity from investors with the purpose of ***selling out-of-the-money hedges options trading at a premium in the market***, as these are used for risk management purposes.
- model risk related to ***callability*** features inserted for the purpose of yield enhancement
- (FI) callable range accruals hedged by digital swaption streams, (EQ) autocallables and cliquets, (FX) power-reversal dual currency options or target redemption forwards,...
- Risk magazine reports that Q4 of 2019 (even before covid), a ***$70bn notional*** of range accrual had to be unwound by the industry

Darwinian principles

- The better models are the ones that are econometrically more realistic and require less frequent and smaller recalibrations over time
- In order for a bad model to remain in use for a long time and affirm itself as a market standard, it ought to satisfy two conditions
- The first one is related to competitive pricing:

  First Darwinian Principle *A lower-quality model surviving the test of time must over-value the structured product at inception.*

- The initial over-pricing results in systematic time-decay of valuations as the model is re-calibrated through time
- The first Darwinian principle raises a profitability puzzle: how the systematic alpha-leakage of a model can be sustained over long time horizons?
- **Second Darwinian Principle** *A surviving lower-quality model must over-hedge and harvest profits from excessive hedge positions which surpass the systematic losses engendered by the initial over-valuation, in the short to medium term.*

VaR, Expected Shortfall and Stressed VaR myopia

- Model risk derives from the cumulative effect of daily recalibrations and feeds into the ***first moment*** of returns, giving rise to a negative alpha leakage.
- VaR, Expected Shortfall or Stressed VaR models do not detect alpha leakages because they focus on ***second and higher moments*** of return distributions and on ***short-time horizons***.
- The losses can only be seen by ***using a good model to simulate hedging strategy of the bad model***

Callable range accrual case study

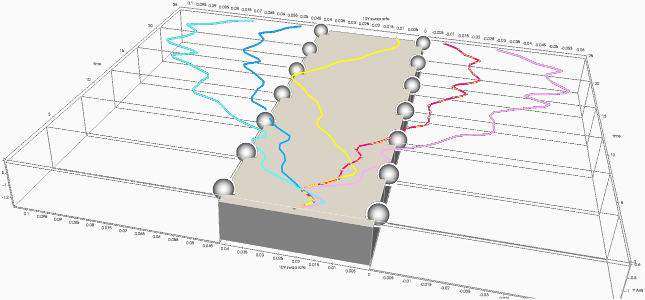

Figure: Range accrual corridor (the grey box), static hedges (on the sides of the corridor), and sample paths.

- Champion, a one-factor model with deeply negative interest rates
- Challenger, a two-factor model with mostly positive interest rates and a stochastic short term long term skew

Seen from the Champion viewpoint:

- large positive gamma of the range accrual
  - $\rightarrow$ easy to hedge
- valuable zero floors (digital swaptions struck at 0) that you have to write
- a dream trade?

That's what model risk is about...

Seen from the Challenger (3D) viewpoint:

- the product should be called when the model hits 0, which is delayed by the Champion: after typically 10 years (over 30) by the Challenger, vs. 20 by the Champion
- Champion implying higher hedge ratios and higher prices than Challenger

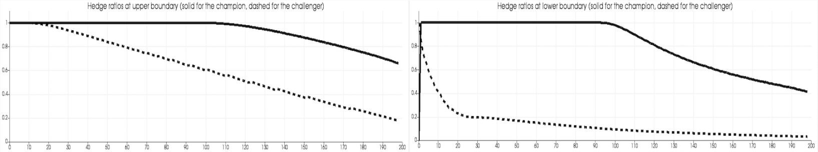

Figure: Comparison of the hedge ratios for Champion and Challenger.

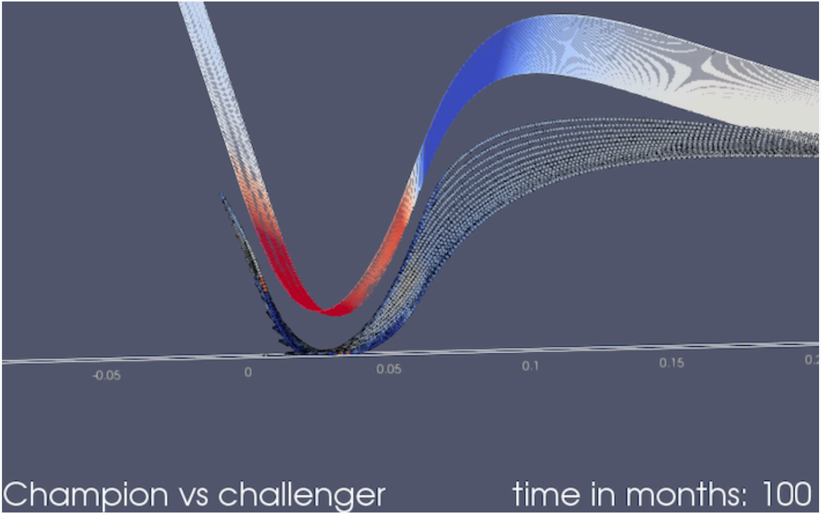

Figure: Comparison between the valuations of Challenger and Champion, color-coded for the gamma (red for positive gamma, white for intermediate, blue for negative).

"Hairs" pointing in the direction of the lowest, negative cross-gamma eigenvalues, telling you how to hedge in the Challenger two-factor model

- directions dangerous for hedging as delta hedging then goes against you

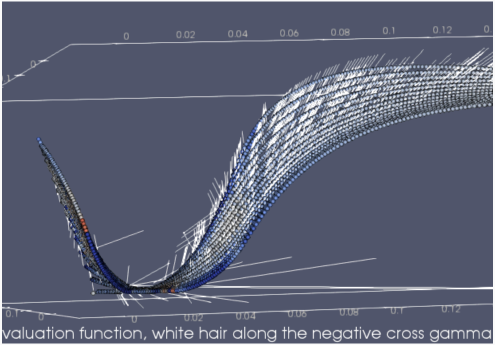

Figure: Valuations of Challenger. The white hairs point in the direction of the negative crossgamma eigenvector.

When rates go to 0, the swaptions that you wrote appreciate and you end up with a large loss

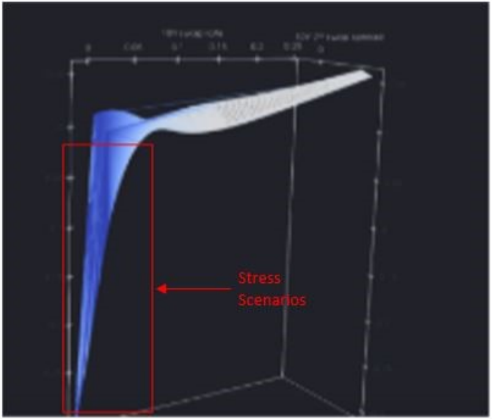

Figure: State space plot for the valuation of excessive hedges shortened for a callable range accrual priced with the Champion (blue for neg. values, white for intermediate).

Accounting Model risk

- ***Accounting methods are models*** in the sense of SR-11-7 because they produce numbers, are based on assumptions, and have an impact on strategies
- ***Model risk*** is a concept that does not apply only to pricing models but extends to accounting methods.

XVA metrics at the test of the Covid-19

- The 2008 crisis triggered a shift from trade-specific pricing to ***netting-set* CVA** analytics
- The 2013-2016 XVA debate revolved around the definition of suitable **FVA** (cost of funding) and **KVA** (cost of capital) metrics
- https://www.**bis**.org/topic/coronavirus.htm :

  The coronavirus (Covid-19) pandemic is a major disruptive event for the global economy. It is revealing financial vulnerabilities and testing the post-financial crisis economic system.
- the Covid-19 financial crisis clearly revealed the inappropriateness of using netting-set aggregation typical to CVA analytics in an FVA and KVA context that requires ***broader funding-set metrics***

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Basle Committee on Banking Supervision. Principles for sound stress testing practices and supervision. Technical report, ***Bank for International Settlements*** (2009):

"Reverse stress tests start from a known stress test outcome (such as breaching regulatory capital ratios, illiquidity or insolvency) and then asking ***what events could lead*** *to such an outcome for the bank."*

We view reverse stress testing as a third stage of evolution for mathematical methods in finance:

- pricing is about the calculation of (risk-neutral) averages,
    - cost of replication
- risk measures calculate tail-conditional expectations of the losses that arise due to imperfect hedging,
- reverse stress testing focuses on individual scenarios and their impact on risk measures.

- Typical model risk methodologies are backward looking
  - ”what went bad”, post mortem time series analysis
  - ”why fix it if it’s not broken”
- Instead, one should anticipate

→ We propose a reverse stress testing methodology based on ***forward-looking simulations***

- Need an actionable metric in this context
- Ultimately one has to maximize shareholder value. Even funding strategies with debt are targeted to this ultimate objective.

→ The perfect optimization target is the present value of future dividend streams, i.e. the ***KVA***

RST Engine

- carry out a portfolio-wide, large scale (possibly nested) simulation
  - 20K outer paths $\times$ 50 time points $\times$ 1K inner paths $\sim$ **1bn simulation nodes**
- compute the time 0 KVA and identify stress scenarios ω as the ones contributing the most to cost of capital, as assessed by ***KVA scenario differentials*** δ_ωKVA.
- drill-down analysis of stress scenarios
  - identify the points in time where the most substantial capital depletions occur
  - gather the vector collecting the corresponding states of each economic factor

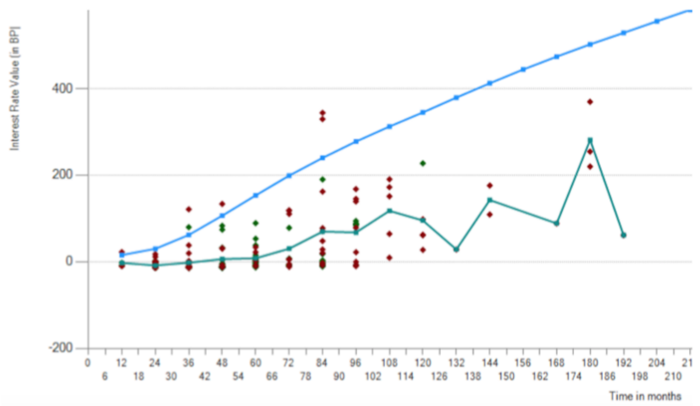

Figure: USD OIS overnight rate (in BPS) for stress scenarios on a large fixed-income derivative portfolio, using Challenger as interest rate model. The blue and green lines represent the forward curve and the forward curve conditional to stress scenarios (average of the dots), respectively.

- systemic vs. idiosyncratic stress scenarios
- ***hedges*** for idiosyncratic stress scenarios
  - long-run hedges that reduce cost of capital, quite different in spirit from short-term delta hedges
- ***trading limits*** based on the counterparty incremental KVA, much more risk sensitive than the counterparty potential future exposure (PFE)
  - fully collateralized derivative portfolios result in a zero PFE although there is collateral funding risk
- ***liquidity risk management*** by shifting from the KVA to the FVA reference metric

Connecting the (model risk and RST) dots

- two co-calibrated models may be very different in the tails

$\rightarrow$ RST fragile to model risk, including accounting practices

- this can be leveraged to the model selection process, for invalidating econometrically unrealistic models in the first place and not using their hedges any at all
- complementing the state-space analysis of the first part of the talk by a ***quantification of the probabilities*** of the extreme risk events

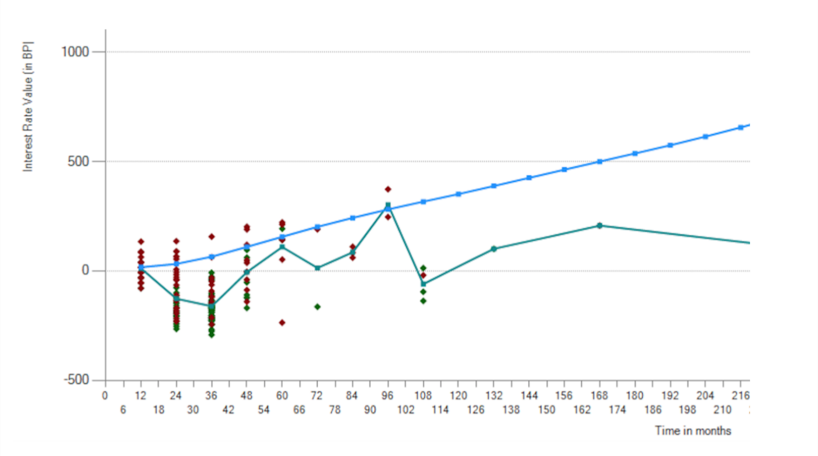

Figure: Analog of the previous figure using Champion as interest rate model: deep negative rates in the first years attract most of the risk.

Bayesian averaging

- Risk capital and KVA should be adjusted to account for the model uncertainty among a family of econometrically realistic and co-calibrated models
- A relevant method is to sample scenarios from the various models, resulting in a meta-model with fatter tails and inflated risk capital numbers

Hoeting, J. A., D. Madigan, A. E. Raftery, and C. T. Volinsky. Bayesian model averaging: A tutorial. *Statistical Science 14*(4), 382–417 (1999).

Kolm, P. and G. Ritter. On the Bayesian interpretation of Black–Litterman. *European Journal of Operational Research 258*(2), 564–572 (2017)

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- Regulation of derivatives $\supseteq$ ***accounting methods***, e.g. specification of XVA metrics of banks
- A crucial modeling issue in finance today, especially in terms of ***model risk***
- Good accounting models for derivatives should involve calculations at the bank's ***overall portfolio*** level, as opposed to netting set by netting set as banks do

Applications of a ***forward-looking RST simulation approach*** include

- long-term hedges of (idiosyncratic) extreme scenarios
- trading limits
- liquidity risk management

Reverse stress testing provides a key constraint on ***model validation***:

- a valid model should (not be subject to large recalibrations and) remain realistic and applicable in all stress scenarios

A further ***management constraint*** derives from the Darwinian principles and is meant to prevent blow-ups and large losses due to over-hedging:

- switching from one model to another whose quality is not clearly better than the first, is allowed only if the second model implies lower valuations for the derivatives being replicated
- equivalently, apply very conservative, risk adjusted, model risk reserves, e.g. of 3 for an uptick of 1

The ***model risk*** inherent to RST can be handled by Bayesian averaging over valid models.