Systemic risk in derivative markets: a graph-theory analysis

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Objectives

• Empirical study on systemic risk in derivative markets
• Integration as a necessary condition for systemic risk to appear
• Previous literature on integration
• Three-dimensional approach
  - Observation time
  - Spatial integration
  - Maturity
• Influence of physical as well as derivative markets
Selected markets

• Choice directed by:
  - Concerns about speculation in commodities
    Energy products
  - Development of bio fuels
    Agricultural products
  - Portfolio management / Commodities as a new class of assets
    Financial instruments
  - Markets with the highest transaction volumes

• 14 markets ( > 655 000 daily futures prices (settlement))
Methodology

• Huge volume of data
• 3 dimensional analysis : complex evolving system
• Use of methods originated from statistical physics
• Graph-theory
• Full connected graph :
  All possible connections between N nodes
• Filtered graph : Minimum Spanning Trees (MST)
  The shortest path between the nodes

1. Construction of MST
2. Topology of the filtered networks
3. Evolution of the MST over time

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1. Minimum spanning trees

- Full connected graph:
  - Synchronous correlation coefficients $\rho$ of prices returns $r$
  - Non linear transformation of correlations into distances:
    $$d_{ij} = \sqrt{2(1 - \rho_{ij})}$$

- Minimum spanning trees:
  - All the nodes of the graph are spanned, with no loop
  - Shortest path linking all nodes
  - Links of the MST are a subset of the initial graph: information space is reduced, from $(N(N-1)/2)$ to $(N-1)$

- Easiest path for the transmission of prices shocks
2. Topology of the MST

- Maturity, spatial and 3-D
- Quantify the ordering of the links
  - Allometric coefficient: $1 < \eta < 2$
    - $\eta = 1$: star tree
    - $\eta = 2$: chain tree
Maturity dimension

Heating oil – Month 1 to 18

Samuelson effect

$\eta = 1.9$

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Spatial dimension : $\eta = 1.493$

Agriculture
- Wheat
- Corn
- Soy Bean
- Soy Oil
- S&P 500

Finance
- Interest rates
- Exchange rates
- Gold

Energy
- Crude, US
- Crude, UK
- Heating oil, US
- Gas oil, UK
- Nat. Gas, US
- Nat. Gas, UK

Commodities:
- Wheat
- Corn
- Soy Bean
- Soy Oil
- Crude, US
- Crude, UK
- Heating oil, US
- Gas oil, UK
- Nat. Gas, US
- Nat. Gas, UK
- S&P 500
3. Dynamical studies

3.1. Correlation coefficients
3.2. Node’s strength
3.3. Normalized tree’s length
3.4. Pruning the trees

Rolling time windows : 480 trading days
3.1. Mean correlations and their variances (3-D)
3.2. Node’s strength

- Full connected graph
- The node’s strength $S_i$ indicates the closeness of one node $i$ to the others:

$$S_i = \sum_{i \neq j} \frac{1}{d_{ij}}$$
Energy

US

EU

Agriculture

Finance

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3.3. Normalized tree’s length

- Sum of the lengths of the links belonging to the MST:

\[ L(t) = \frac{1}{N-1} \sum_{(i,j) \in MST} d_{ij} \]

- The more the length shortens, the more integrated the system is

Spatial dimension
3.4. Pruning the trees

- Analysis of inter-market and inter-sectors reorganizations
- Consider only the links between markets, whatever the maturity is considered
- Survival ratios on the basis of market links, in the MST
  - Robustness of the topology over time
  - The survival ratio $S_R$ refers to the fraction of links that survives between two consecutive trading days:

$$S_R(t) = \frac{1}{N-1} \left| E(t) \cap E(t-1) \right|$$

$E(t)$ : set of links at date $t$
Pruned trees, survival ratios

\[ S_R \]

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Main results & conclusions

1. Topology
   - Chain-like trees in the maturity dimension
   - Star-like trees in the spatial and 3-D dimensions

2. Emerging taxonomy
   - Trees organized around the three sectors of activity
   - Center of the graph: two crude oils

3. Integration
   - Increases in all dimensions (spatial, maturity, 3D)
   - Progresses at the heart of the system

4. Next
   - Analysis of price’s shocks
   - Causality and directed graphs
   - Adding new information in the graph

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